12) $A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$ $\quad P(\lambda) = \det\left(\begin{bmatrix} \lambda-1 & -2 \\ -2 & \lambda-1 \end{bmatrix}\right) = \lambda^2 - 2\lambda - 3$

Autoval. $\to \underbrace{\lambda^2 - 2\lambda - 3 = 0}_{\frac{2 \pm \sqrt{4+12}}{2}} \to \lambda_1 = 3$

$\to \lambda_2 = -1$.

Para $\lambda = 3$

$\begin{pmatrix} 2 & -2 \\ -2 & 2 \end{pmatrix}_{F2 \to F1+F2} \begin{pmatrix} 2 & -2 \\ 0 & 0 \end{pmatrix} \to 2x - 2y = 0 \to x = y$.

$\bar{X} = x \cdot \underbrace{(1,1)}_{\text{AUTOVECT. } \lambda = 3}$.

Para $\lambda = -1$

$\begin{pmatrix} -2 & -2 \\ -2 & -2 \end{pmatrix}_{F2 \to F1-F2} \begin{pmatrix} -2 & -2 \\ 0 & 0 \end{pmatrix} \to -2x - 2y = 0 \to x = -y \to y = -x$.

$\bar{X} = x \cdot \underbrace{(1,-1)}_{\text{AUTOVECT. } \lambda = -1}$.

$Y_1 = e^{3t} \cdot \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ $\quad Y_2 = e^{-t} \cdot \begin{pmatrix} 1 \\ -1 \end{pmatrix}$

$$Y(t) = k_1 \cdot e^{3t} \cdot \begin{pmatrix} 1 \\ 1 \end{pmatrix} + k_2 \cdot e^{-t} \cdot \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

Como $Y(0) = Y_0 = (a, 6)$

$$\rightarrow k_1 \cdot \begin{pmatrix} 1 \\ 1 \end{pmatrix} + k_2 \cdot \begin{pmatrix} 1 \\ -1 \end{pmatrix} = \begin{pmatrix} a \\ 6 \end{pmatrix}$$

$\rightarrow k_1 + k_2 = a \rightarrow k_1 = a - k_2 \rightarrow k_1 = a - \left(\frac{a-6}{2}\right) = \frac{a+6}{2}$.

$k_1 - k_2 = 6 \rightarrow a - 2k_2 = 6 \rightarrow k_2 = \frac{6-a}{-2} = \frac{a-6}{2}$

$$\rightarrow Y(t) = \underbrace{\left(\frac{a+6}{2}\right) \cdot e^{3t} \cdot \begin{pmatrix} 1 \\ 1 \end{pmatrix}} + \left(\frac{a-6}{2}\right) \cdot e^{-t} \cdot \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

Para que $\|Y(t)\|$ sea acotada, entonces cuando $t \rightarrow \infty$

$\frac{a+6}{2} = 0 \rightarrow a + 6 = 0 \rightarrow a = -6$.

Por lo tanto los valores de $Y_0 \in \mathbb{R}^2$ que cumplen esto son de la forma $(-6, 6)$